Robust Coefficients Alpha and Omega and Confidence Intervals: Methods and Software

Zhiyong Zhang and Ke-Hai Yuan
University of Notre Dame

Abstract

Cronbach's coefficient alpha is a widely used reliability measure in social, behavioral and education sciences. It is reported in nearly every study that involves measuring a construct through multiple items. For non-tau-equivalent test, McDonald's omega has been used as a popular alternative to alpha in the literature. Traditional estimation methods for alpha and omega often implicitly assume that data are complete and normally distributed. This study proposes robust procedures to estimate alpha and omega as well as corresponding standard errors and confidence intervals from samples that may contain potential outlying observations and missing values. The influence of outlying observations and missing data on alpha and omega is investigated through two simulation studies. The simulation results show that the newly developed robust method yields substantially improved alpha and omega estimates as well as coverage rates of confidence intervals than the conventional non-robust method. An R package coefficientalpha is developed and demonstrated to estimate both robust alpha and omega.

Keywords: Robust Cronbach's alpha, robust McDonald's omega, missing data, outlying observations, confidence intervals, R package coefficientalpha

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Correspondence can be sent to Zhiyong Zhang, Department of Psychology, University of Notre Dame, 118 Haggar Hall, Notre Dame, IN 46556. This research is partially supported by a grant from the Department of Education (R305D140037). However, the contents of the paper do not necessarily represent the policy of the Department of Education, and you should not assume endorsement by the Federal Government.

Introduction

Coefficient alpha (α, also referred to as Cronbach's alpha, Cronbach, 1951, 1988; Cronbach & Shavelson, 2004) is a widely used measure of reliability in social, behavioral and education research although there are oppositions to the use of it (e.g., Green & Yang, 2009a; Sijtsm, 2009). Sample coefficient alpha is reported in nearly every study involving the measure of a construct through multiple items in psychological research because of its historical popularity, ease of calculation, and availability in statistical software. For tau-equivalent test, sample coefficient alpha is a consistent estimator of the reliability of the test; otherwise, sample coefficient alpha typically underestimates the reliability (e.g., Maydeu-Olivares et al., 2010; Raykov, 1997, 2001). When measurements are not tau-equivalent, McDonald's omega (ω) has been recommended to measure reliability for homogeneous items (McDonald, 1999).

In addition to reporting the sample coefficient alpha and omega, many researchers strongly recommended reporting corresponding standard errors (SE) and/or confidence intervals (CIs, e.g., Fan & Thompson, 2001; Iacobucci & Duhachek, 2003; Raykov & Shrout, 2002). There also exist developments regarding the SEs or the sampling distributions of the sample coefficient alpha and omega under various conditions. For example, under the assumption of parallel items and normal data, Kristof (1963) and Feldt (1965) showed that a transformation of the sample coefficient alpha follows an F distribution. Under the multivariate normal assumption, van Zyl, Neudecker, and Nel (2000) showed that the sample coefficient alpha asymptotically follows a normal distribution without assuming compound symmetry. Yuan and Bentler (2002) further found that the asymptotic distribution given by van Zyl et al. can still be valid for a large class of nonnormal distributions. Maydeu-Olivares et al. (2007) and Yuan et al. (2003) studied the distributional properties of sample coefficient alpha, and their results are asymptotically valid for all populations with finite fourth-order moments. There are fewer studies on the sampling distribution of omega in the literature (Cheung, 2009; Raykov, 2002; Yuan & Bentler, 2002). For example, Raykov (2002) proposed an analytical procedure that can estimate the standard error of sample omega. Yuan and Bentler (2002) developed a method that yields the robust standard error for sample omega when data are non-normal.

However, previous development on the estimation of alpha and omega as well as corresponding standard errors and confidence intervals rarely considers the influence of outlying observations and missing data. The estimation of alpha and omega is typically based on the sample covariance matrix, which is extremely sensitive to outlying observations. Therefore, it is expected that alpha and omega are equally influenced by outlying observations. In fact, previous literature has shown that the sample coefficient alpha can be biased or very inefficient with the presence of outlying observations and non-normal data (e.g., Liu & Zumbo, 2007; Liu, Wu & Zumbo, 2010; Sheng & Sheng, 2013). Enders (2004) showed that ignoring missing data in the sample might lead to biased and less accurate estimate of coefficient alpha and proposed an expectation-maximization algorithm to deal with missing data for estimating reliability (see also, Enders, 2003). Headrick and Sheng (2013) proposed to estimate alpha based on the L-moment to handle non-normal data. However, methods to deal with both outlying observations and missing data in estimating alpha, especially omega, are desired.

The purpose of this study is two-folded. The first is to propose robust M-estimators of alpha and omega. We will obtain the robust estimators as well as their standard errors and CIs for alpha and omega to deal with both outlying observations and missing data. The second is to develop easy to use software (R package coefficientalpha) that yields robust estimates of alpha and omega as well as the corresponding confidence intervals. The robustness of the developed procedure relies on the fact that outlying cases are downweighted in the estimation process. Missing data are handled by an expectation-robust (ER) algorithm (Yuan, Chan & Tian, 2015), which is a generalization of the EM-algorithm based on a multivariate t-distribution (Little, 1988). In the development, we assume that data are either missing completely at random (MCAR) or missing at random (MAR; e.g., Little & Rubin, 2002). As we shall see, the software not only provides the calculation of robust coefficient alpha and omega and their consistent standard errors, it also provides diagnostic plots for visually examining cases that are influential to the estimation of alpha and omega. Other desirable features of the software include being free, and being able to run either locally on a personal computer within the statistical software R or remotely on a web server. In particular, our user-friendly web interface does not require researchers to be familiar with R to use the software.

In the next section, we first distinguish two types of outlying observations and show their influence on coefficient alpha. Then, we describe how to obtain robust coefficient alpha and the corresponding CI. Next, we show how to apply the robust procedure to estimate omega and its confidence intervals. After that, we illustrate the influence of outlying observations and missing data on alpha and omega through two simulation studies. Through the simulation studies, we also show that the newly developed robust method yields substantially improved alpha and omega estimates as well as coverage rates of confidence intervals than the conventional non-robust method. Finally, we demonstrate the use of our software through an example. Some technical details are provided in the appendix.

Outlying Observations and Their Influence

We first illustrate the influence of outlying observations on coefficient alpha through an example. Using the example, we also explain the concept of outlying observations. The influence of outlying observations on omega is similar as we will show in our simulation studies.

Let $\mathbf{y}$ be a vector that denotes a p-variate population with mean $\boldsymbol{\mu}$ and covariance matrix

$\boldsymbol{\Sigma} = (\sigma_{ij})$. The population coefficient alpha for the summation of the scores in $\mathbf{y}$ is defined as

$$\alpha = \frac{p}{p-1}\left(1 - \frac{\sum_{i=1}^{p} \sigma_{ii}}{\sum_{i=1}^{p}\sum_{j=1}^{p} \sigma_{ij}}\right). \tag{1}$$

Let $\mathbf{y}_1, \ldots, \mathbf{y}_n$ be a random sample of $\mathbf{y}$ and $\mathbf{S} = (s_{ij})$ be the sample covariance matrix. Then the sample coefficient alpha is given by

$$\hat{\alpha} = \frac{p}{p-1}\left(1 - \frac{\sum_{i=1}^{p} s_{ii}}{\sum_{i=1}^{p}\sum_{j=1}^{p} s_{ij}}\right). \tag{2}$$

The influence of outlying observations on sample coefficient alpha can be clearly illustrated through the analysis of two items. Here, we demonstrate the influence numerically through an example.

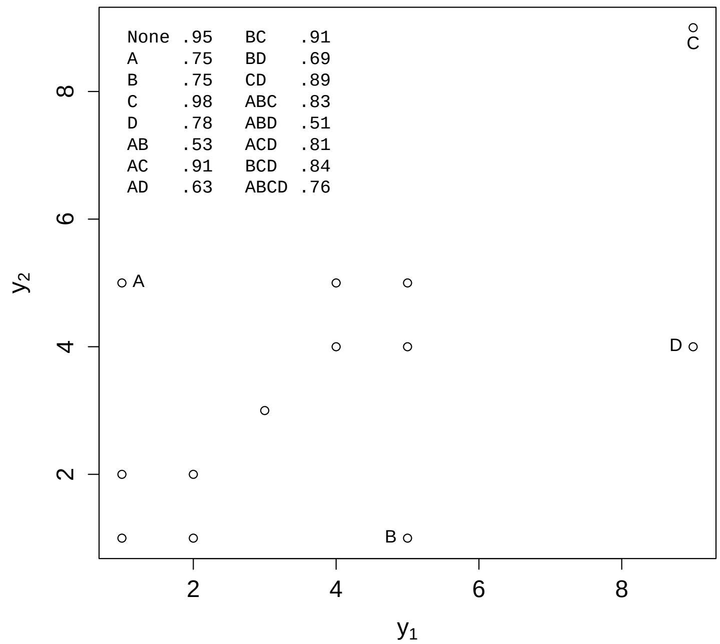
Figure 1 displays a data set of 13 observations on two items y_1 and y_2 with different types of outlying observations. The coordinates of the 9 regular observations are given by

$$\begin{array}{cccccccccc} y_1 & 1 & 1 & 2 & 2 & 3 & 4 & 4 & 5 & 5 \\ y_2 & 1 & 2 & 1 & 2 & 3 & 4 & 5 & 4 & 5. \end{array}$$

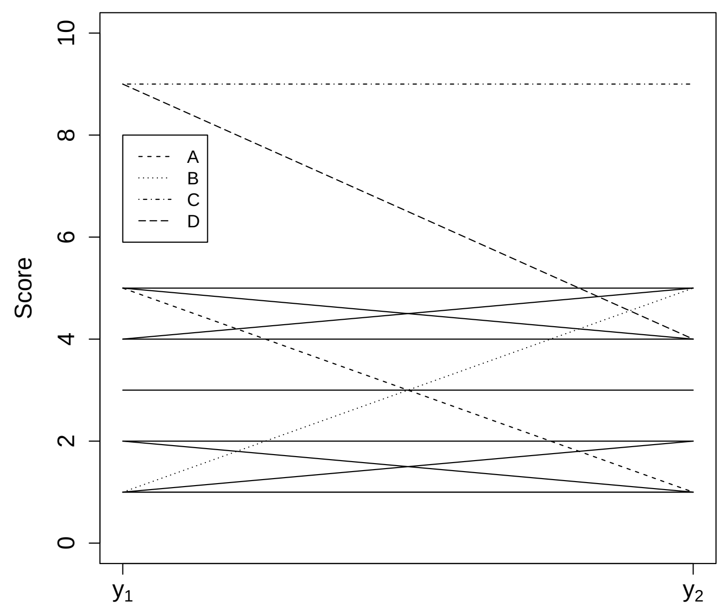
The coordinates of the 4 additional observations are A$= (1, 5)$, B$= (5, 1)$, C$= (9, 9)$ and D$= (9, 4)$. Figure 1 also contains the values of coefficient alpha when different combinations of A, B, C, and D are pooled with the 9 regular observations. It follows from the geometry of the 9 regular observations that A, B, C, and D are outlying observations. The 4 observations can then be classified according to their locations or geometry relative to the other cases.

We first distinguish between inadmissible outlying observations and admissible outlying observations. Inadmissible outlying observations are typically erroneous observations that do not represent the underlying phenomena to be measured. The scores for inadmissible outlying observations can be either within or out of the permissible range of a test. Data recording and input error is the most common cause of inadmissible outlying observations. For example, suppose that y_1 and y_2 are 5-point Likert items. Then, C and D in Figure 1 are obviously erroneous observations because they have scores larger than 5. Generally speaking, our robust procedure can be used to effectively detect inadmissible observations but does not automatically remove them. Once such observations are detected, special treatment should be taken by substantive researchers before conducting any serious data analysis, including the estimation of reliability measures.

Admissible outlying observations are different from the majority of the data but carry valid and useful information and truly represent the underlying phenomena. For example, suppose both y_1 and y_2 are continuous measures of mathematics ability with the maximum possible score 10. For illustration purpose, we further assume that y_1 is a measure of addition and y_2 is a measure of multiplication. Reasonably, a theory can be hypothesized that the two items are determined by the



(a) Scatterplot



(b) Profile plot

Figure 1. Diferent types of outlying observations and their influence on coefficient alpha

same unidimensional underlying mathematical ability and should be positively correlated. In this case, the four observations A, B, C, and D do have valid scores but they also appear different (outlying) from the rest of the observations.

For the four outlying observations, each of them represents a different pattern. First, the observation C has extreme scores on both y_1 and y_2 but within the limit of maximum scores. Furthermore, the two scores are extreme in the same direction and therefore consistent with the hypothesized theory. The observation is outlying mostly likely because of the participant's talent in mathematics that leads to large scores for y_1 and y_2. In this study, we refer to such outlying observations that are consistent with the hypothesized theory as leverage observations (Yuan & Zhong, 2008). In general, leverage observations are "good" in the sense that they lead to enlarged coefficient alpha. For example, when the 9 regular observations are considered, the coefficient alpha is 0.95. With C being included, the alpha increases from 0.95 to 0.98.

The observations A, B, and D are also outlying but geometrically different from C. For example, for A, it has a large score on y_2 but a small score on y_1. In other words, the observation shows high ability in terms of multiplication but low ability on addition. This is inconsistent with the hypothesized theory on the unidimensional mathematical ability. The pattern of the observation B is the other way around. Although D has a relative large score on y_1, its value is a lot smaller compared to that of y_2. In this study, we refer to such outlying observations that are inconsistent with the hypothesized theory as outliers (Yuan & Zhong, 2008). Outliers typically reduce the value of coefficient alpha. For example, with A or B being included, the coefficient alpha changes from 0.95 to 0.75 and with D being included, the alpha changes to 0.78.

In practice, a data set may contain both outliers and leverage observations; whether the estimated coefficient alpha becomes smaller or larger depends on their combined effects. For example, with all three outliers A, B, and D, the estimated coefficient alpha changes from the original 0.95 to 0.51. With outlier A and leverage observation C, the estimated coefficient alpha has a small decrease to about 0.91. However, in general, we may not want the estimated coefficient alpha to be determined by a few outlying observations, regardless of outliers or leverage observations. This motivates our development of a robust estimator of coefficient alpha.

Although the scatterplot is useful in identifying outlying observations with two items, it can be difficult to use when there are more than 2 items. Instead, a profile plot can be used to visualize possible outlying observations (e.g., Yuan & Zhang, 2012b). For example, the profile plot in Figure 1b displays the same data as in the scatterplot. It also shows the difference between the regular and outlying observations as well as that between outliers and leverage observations. First, the outlying observations distinguish themselves from the regular observations because their profiles do not mingle together with those of regular observations. Second, for outliers, their profiles usually show atypical patterns. For example, the profiles of A and B show more change than regular ones by having high score on one item but low score on the other item. Third, for leverage observations, all items may have scores noticeably smaller or larger than those in other cases, and their profiles, such as observation C, separate themselves from the profiles of the rest of cases. Note that the profile of D exhibits patterns of both outliers and leverage observations.

Robust Coefficient Alpha

In this section we will first review robust estimation of covariance matrix with complete data, and then describe the idea of robust estimation with incomplete data. Robust estimate of the coefficient alpha will be defined next. We will also discuss how to choose a robust estimator.

The idea of robust covariance matrix is to use a formula in which cases that are unusually far from the center of the majority of observations get smaller weights and, therefore, contribute less to the covariance estimates. For complete data, a robust covariance matrix can be estimated according to

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^{n} w_{i2}(\mathbf{y}_i - \hat{\boldsymbol{\mu}})(\mathbf{y}_i - \hat{\boldsymbol{\mu}})', \tag{3}$$

where

$$\hat{\boldsymbol{\mu}} = \frac{1}{\sum_{i=1}^{n} w_{i1}} \sum_{i=1}^{n} w_{i1}\mathbf{y}_i \tag{4}$$

is the corresponding robust mean estimate; w_{i1} and w_{i2} are weights that are inversely proportional to the deviation between $\mathbf{y}_i$ and $\hat{\boldsymbol{\mu}}$, as measured by the Mahalanobis distance (M-distance; see Yuan & Zhang, 2012b). Because a larger distance corresponds to a smaller weight, cases that are far from the center of the majority of observations (represented by $\hat{\boldsymbol{\mu}}$) will have a small or tiny contribution to the estimator $\hat{\boldsymbol{\Sigma}}$ in Equation 3. Thus, the effect of outlying observations on $\hat{\boldsymbol{\Sigma}}$ is limited. Notice that, Equation 3 yields the sample covariance matrix $\mathbf{S}$ when $w_{i1} = w_{i2} = 1$ for all i. Because outlying cases are treated equally as regular cases in the formulation of $\mathbf{S}$, outlying observations have unlimited influences on $\mathbf{S}$ and consequently on the resulting coefficient alpha.

Both outliers and leverage observations will deviate more from the center of the majority of data than other typical observations. Their effect on $\hat{\boldsymbol{\Sigma}}$ can be controlled when w_{i1} and w_{i2} are properly chosen. Many candidates of weights have been suggested in the robust statistical literature. The Huber-type weights are widely used and are therefore employed in the current study (e.g., Yuan & Zhang, 2012a).[1] In Huber-type weights, a tuning parameter φ $(0 \leq \varphi < 1)$ is used to practically control the percentage of data to be downweighted. For example, the tuning parameter is used to determine a threshold for the calculation of the weights. If the distance of a case from $\hat{\boldsymbol{\mu}}$ is less than the threshold, the case is not downweighted; and otherwise, the case will be assigned a weight that is smaller than 1 in Equation (3) and its value is inversely proportional to the distance. When $\varphi = 0$, no case is downweighted. With the increase of φ, more cases will

[1]The Huber-type weights also rescale the resulting $\hat{\boldsymbol{\Sigma}}$ to be unbiased under normality. But the rescale factor does not affect the estimated alpha and omega.

be downweighted and cases are also more heavily downweighted. A larger φ would make alpha more resistant to outliers but could also lose efficiency when a substantial proportion of the observations that truly represent the population are downweighted. In practice, a weight between 0.01 and 0.1 would work well in controlling the effect of outlying observations on the estimate of coefficient alpha. We will discuss how to choose a proper φ with graphs and examples shortly.

The idea of robust estimation with missing values is the same as robust estimation with complete data. The algorithm for calculating the robust estimates of means and covariance matrix is called the expectation robust (ER) algorithm. In the E-step, terms involving missing values in Equation 3 are replaced by their conditional expectations, and the R-step parallels to that in Equation 3, where cases that sit far from the center of the majority of observations will be downweighted. In our software, the implementation of the ER-algorithm with the Huber-type weights is based on Yuan et al. (2015).

With the robust covariance matrix $\hat{\mathbf{\Sigma}} = (\hat{\sigma}_{ij})$, the robust alpha estimate is

$$\hat{\alpha} = \frac{p}{p-1}\left(1 - \frac{\sum_{j=1}^{p}\hat{\sigma}_{jj}}{\sum_{i=1}^{p}\sum_{j=1}^{p}\hat{\sigma}_{ij}}\right). \tag{5}$$

In addition to the point estimator $\hat{\alpha}$, the output of the software also contains the SE of $\hat{\alpha}$ and the corresponding CI for alpha. The SE is based on the sandwich-type covariance matrix for the robust estimate $\hat{\mathbf{\Sigma}}$, and is consistent regardless of the data distribution (Yuan et al., 2015). It can also provide us the information on the efficiency of the robust method. The details of the ER-algorithm and the sandwich-type covariance matrix for $\hat{\mathbf{\Sigma}}$ are not the focus of the study, and are referred to Yuan et al. (2015) and Yuan and Zhang (2012a). The SE for $\hat{\alpha}$ based on the sandwich-type covariance matrix of $\hat{\mathbf{\Sigma}}$ is provided in Appendix A.

Because the robustness is determined by the tuning parameter φ, we next describe three graphs to facilitate its choice and display its effect.

The first one is termed as a diagnostic plot that displays the values of coefficient alpha against different values of φ. For a given data set, alpha will generally change when φ varies. For example, if there are outliers, alpha would first increase quickly and then flatten out with the increase of φ. If there are leverage observations, alpha will decline as φ increases. When only outliers exist, one may choose an "optimal" φ according to the maximum value of alpha or at the place where alpha shows the largest changes. When only leverage observations exist, one may choose the tuning parameter φ that corresponds to the first substantial drop of alpha to control the effect of leverage observations. When leverage observations and outliers coexist, one may slowly increase the tuning parameter φ to choose its value so that alpha becomes stabilized.

The second graph is termed as a weight plot where case-level weights w_{i2} against case ID are plotted. We do not include the plot of w_{i1} because it contains the same information as the plot of w_{i2}. In the plot, cases with smallest weights are identified by case numbers. The plot provides the information on which cases are outlying and how heavily they are downweighted.

The third graph is a profile plot in which centered observations $(y_{ij} - \hat{\mu}_j)$ for cases with smallest weights are plotted against the order of the variable j. With such a plot, one can visually examine how the profile of each outlying case is different from the average profile and therefore, why a case is downweighted. The profile plot also provides information on whether an outlying case is an outlier or a leverage observation.

In summary, the three plots together will allow us to select a proper φ according to the distribution shape of the data. They also facilitate us to see how coefficient alpha change when controlling the effect of outliers and/or leverage observations. We will illustrate how to use the graphs in practice later on in an example when introducing our software.

Robust Coefficient Omega

The procedure for obtaining robust coefficient alpha can be readily extended to omega for homogenous items. Suppose that p measurements follow a 1-factor model

$$y_{ij} = \mu_j + \lambda_j f_i + e_{ij}$$

where y_{ij} denotes a score for participant i on item j, μ_j is the intercept for item j, f_i is the common factor score for participant i, λ_j is the factor loading for item j, and e_{ij} is the independent unique factor score with variance ψ_{jj}. The variance of f_i is fixed at 1.0 for model identification. The population omega (ω) is defined as (McDonald, 1999, Equation 6.20b),

$$\omega = \frac{(\sum_{k=1}^{p} \lambda_j)^2}{(\sum_{k=1}^{p} \lambda_j)^2 + (\sum_{k=1}^{p} \psi_{jj})}.$$

To estimate ω, the factor model is typically estimated from the sample covariance matrix of $\mathbf{y}$. Therefore, the sample omega, denoted by $\hat{\omega}$, is similarly influenced by outlying observations as the sample coefficient alpha. Actually, except in rare cases, the difference between $\hat{\omega}$ and $\hat{\alpha}$ is small for complete and normally distributed data (e.g., Maydeu-Olivares et al., 2010; Raykov, 1997). Outliers and leverage observations can also be distinguished using the factor model (see Yuan & Zhang, 2012b). Generally speaking, cases with extreme common factor scores are leverage observations and data contamination leads to cases with large unique factor scores or outliers.

The robust $\hat{\omega}$ can be calculated through a multi-stage procedure. First, a robust covariance matrix $\hat{\mathbf{\Sigma}}$ as in the estimation of robust alpha can be obtained considering both outlying observations and missing data. Second, the factor model can be estimated by any SEM software based on the robust $\hat{\mathbf{\Sigma}}$ to get $\hat{\lambda}_j$ and $\hat{\psi}_{jj}$. Third, the robust $\hat{\omega}$ is calculated as

$$\hat{\omega} = \frac{(\sum_{k=1}^{p} \hat{\lambda}_j)^2}{(\sum_{k=1}^{p} \hat{\lambda}_j)^2 + (\sum_{k=1}^{p} \hat{\psi}_{jj})}.$$

In order to get the robust standard error or confidence interval for omega, one first obtains the covariance matrix for $\hat{\boldsymbol{\theta}} = (\hat{\lambda}_1, \ldots, \hat{\lambda}_p, \hat{\psi}_{11}, \ldots, \hat{\psi}_{pp})$ and then uses the delta method to get the standard error for $\hat{\omega}$. Both the standard error of $\hat{\omega}$ and confidence interval of omega are available in our R packages and therefore the technical details are omitted here for the sake of space. The robustness of omega is related to the tuning parameter φ as for alpha. The same method for determining φ for alpha can be applied here.

Simulation Studies

In this section, we present two simulation studies to show the influence of outlying observations and missing data on the estimates of alpha and omega. For the sake of space, we only report the results of a few conditions.

Simulation Study 1: Influence of Outlying Observations

The aim of the study is to demonstrate the influence of outlying observations on the estimates of alpha and omega through simulated data. Consider the one-factor model

$$y_{ij} = \mu_j + \lambda_j f_i + e_{ij}, i = 1, \ldots, n, j = 1, \ldots, p.$$

Two sets of population parameters are used in this study. One satisfies tau-equivalence with $\lambda_j = \sqrt{0.6}$ and $\psi_{jj} = 0.4$ for $j = 1, \ldots, p$ and $p = 6$. Thus, the population alpha and omega are the same and equal to 0.9. Another does not conform to tau-equivalence with $\lambda_j = \sqrt{0.2}$ and $\psi_{jj} = 0.8$ for $j = 1, 2, 3$ and $\lambda_j = \sqrt{0.6}$ and $\psi_{jj} = 0.4$ for $j = 4, 5, 6$. The intercept μ_j is set to 0 without losing generalization. Under this setting, the population alpha is 0.777 and the population omega is 0.789. The alpha is smaller than omega as expected (e.g., McDonald, 1999) but the difference is not substantial.

Data are generated as following. First, 1000 sets of normal data with sample size $n = 100$ are generated from both tau-equivalent and non-tau-equivalent models, respectively. Second, for each normal data set, a new set of data with outliers is created by letting y_{ij} = $y_{ij} - 4$ for $j = 1, 2, 3$ and y_{ij} = $y_{ij} + 4$ for $j = 4, 5, 6$ with $i = 96, 97, 98, 99, 100$. Therefore, each new data set has 5% of outliers. Third, for each normal data set, another new set of data with leverage observations is created by letting y_{ij} = $y_{ij} - 6$ for $j = 1, \ldots, 6$ with $i = 96, 97, 98, 99, 100$. Therefore, each new data set has 5% of leverage observations.

Both alpha and omega are then estimated from the generated data with three different levels of downweighting rate $\varphi = 0, 0.05, 0.1$. Note that when $\varphi = 0$, no data are downweighted and therefore both alpha and omega are estimated by the commonly used non-robust method. Table 1 presents the results as the average of the estimated alpha and omega as well as their empirical standard errors and the coverage rates of the 95% confidence intervals.

Table 1

Average alpha and omega and their empirical standard errors and coverage rates of 95% confidence intervals for Study 1 under normal data (Normal), data with outliers (Outlier), and data with leverage observations (Leverage). The population alpha and omega are both 0.9 for the tau-equivalent model and for the non-tau-equivalent model, the population alpha is 0.777 and the population omega is 0.789.

			alpha			omega		
		φ	Est	s.e.	Coverate	Est	s.e.	Coverage
tau-equivalent	Normal	0	.898	.015	.934	.899	.016	.939
		0.05	.898	.016	.942	.899	.016	.944
		0.1	.898	.016	.950	.899	.016	.946
	Outlier	0	.663	.109	.115	.600	.101	0
		0.05	.863	.047	1	.862	.049	1
		0.1	.872	.033	.991	.873	.033	.995
	Leverage	0	.952	.016	1	.952	.016	1
		0.05	.901	.057	.928	.903	.055	.923
		0.1	.885	.053	.900	.888	.050	.899
non-tau-equivalent	Normal	0	.772	.034	.934	.786	.033	.933
		0.05	.772	.035	.937	.786	.034	.935
		0.1	.772	.036	.939	.786	.034	.930
	Outlier	0	.437	.161	.313	.456	.112	.025
		0.05	.691	.100	1.00	.691	.102	.988
		0.1	.712	.073	.989	.716	.075	.980
	Leverage	0	.914	.024	.861	.916	.024	.996
		0.05	.886	.047	.933	.890	.045	.955
		0.1	.874	.045	.917	.878	.043	.930

Note. Est: estimate; s.e.: Standard error; Coverage: coverage rate of 95% confidence intervals.

Results for the analysis of data from the tau-equivalent model are presented in the upper-panel of Table 1. The following can be concluded from the results. First, for the normal data, the average of the estimated alpha and omega are extremely close, with a difference about 0.001, and are almost the same as the population value. Furthermore, downweighting the normal data does not cause noticeable difference in alpha and omega from the undownweighted ones. In addition, the coverage rates are close to the nominal level 0.95 for both alpha and omega. Second, for the data with outliers, the estimated non-robust alpha is 0.663 and the estimated non-robust omega is 0.600, both of which largely underestimate their population values. It seems that omega is influenced even more by outliers than alpha. The coverage rate for alpha is 0.115 and for omega is 0, both of which largely deviate from the nominal level 0.95. After downweighting, the resulting robust alpha and omega become much more closer to their population values and the estimated standard errors become substantially smaller than their non-robust counterparts. The estimated robust alpha and omega are almost identical to each other. Although the coverage rates are larger than the nominal level 0.95, they are much better than those from the non-robust method. Third, for the data with leverage observations, the non-robust alpha and omega overestimate their population values. After downweighting, the robust alpha and omega become closer to but are still above the population values. However, their standard errors become notably larger. This is because the leverage observations tend to yield more efficient parameter estimates (Yuan & Zhong, 2008) and downweighting minimizes the effect. Before downweighting, the coverage rates are 1, larger than 0.95 and after downweighting, they are smaller than 0.95.

The lower-panel of Table 1 contains the results for the analysis of data from the non-tau-equivalent model, from which we can conclude the following. First, for the normal data, the average of the estimated omega is slightly larger than the average of the estimated alpha, both are very close to their population values. The same phenomenon as for the data from the tau-equivalent model is observed, that is downweighting data does not play an influential role in the estimated alpha and omega for normally distributed samples. Second, for the data with outliers, the estimated non-robust alpha and omega are much smaller than their population values. Although the robust alpha and omega are still smaller than their population values, the improvement is substantial. Third, for data with leverage observations from the non-tau-equivalent model, both non-robust and robust alpha and omega overestimate their population values. The robust ones show substantial improvement over the non-robust ones. The pattern of coverage rates is similar to that of tau-equivalent data.

Concluding Remarks. This simulation study shows that, for normally distributed data, there is not substantial difference between alpha and omega regardless of whether the model is tau-equivalent or not. Outliers cause underestimation of alpha and omega whereas leverage observations cause their overestimation. Our robust procedure can effectively control the influence of outlying observations although the procedure seems to work more effectively for outliers than for leverage observations. In particular, the confidence intervals from our robust method have much improved coverage rates.

Simulation Study 2: Influence of Missing Data

This study aims to demonstrate the influence of missing data on alpha and omega through simulated data. Complete normal data are generated from the tau-equivalent model and the non-tau-equivalent model as in Study 1. Incomplete data are obtained in the following way. First, there are no missing data in y_1 and y_4. Second, missing data in y_5 and y_6 are related to y_1. Specifically, an observation for y_5 is missing if $y_1 \leq q_{0.1}(y_1)$ and an observation for y_6 is missing if $q_{0.1}(y_1) < y_1 \leq q_{0.2}(y_1)$ where $q_p(y)$ is the $100p$th percentile of y. Third, missing data in y_2 and y_3 are related to y_4 so that an observation for y_2 is missing if $y_4 \geq q_{0.9}(y_4)$ and an observation for y_3 is missing if $q_{0.8}(y_4) \leq y_4 < q_{0.9}(y_4)$. According to the definition of missing data mechanisms (e.g., Little & Rubin, 2002), all the missing values are missing at random.

Both alpha and omega are estimated from the generated data with missing values using either listwise deletion or the robust procedure developed in this paper that automatically takes care of missing data. Table 2 presents the results as the average of the estimated alpha and omega as well as their empirical standard errors and the coverage rates of the 95$\%$ confidence intervals.

First, it is clear that the sample alpha and omega following from listwise deletion underestimate their population values for both tau-equivalent and non-tau-equivalent models. Second, the estimated alpha and omega following the robust method are very close to their population values. Third, it is evident that the standard errors of the estimated alpha and omega from the robust method are much smaller than those based on listwise deletion. This is because the robust method utilizes more information in the data. Fourth, when listwise deletion is used, the coverage rates are largely smaller than the nominal level 0.95, especially for the tau-equivalent data. Our robust method, on the other hand, yields coverage rates much closer to 0.95. To separate the influence of nonnormal data and missing data on the reliability estimates, only normal data are investigated in this study. We also have evidences that for nonnormal data, the influence of missing data is similar.

Software

To facilitate the calculation of robust alpha and omega, we developed an R package coefficientalpha. The package is freely available at `http://cran.r-project.org/package=coefficientalpha`. The package calculates the robust alpha and omega, their standard errors, and confidence intervals for a given data set. In addition, the package also generates the diagnostic plot, the weight plot and the profile plot to assist the selection of the tuning parameter and to visualize outlying observations. To accommodate the users who are not familiar with R, an online interface is also developed. The R

Table 2

Average alpha and omega and their empirical standard errors and coverage rates of the 95% confidence intervals for Study 2 under listwise deletion (Deletion) and maximum likelihood method (ML). The population alpha and omega are both 0.9 for the tau-equivalent model and for the non-tau-equivalent model, the population alpha is 0.777 and the population omega is 0.789.

			alpha			omega		
		φ	Est	s.e.	Coverage	Est	s.e.	Coverage
tau-equivalent	Deletion	0	.804	.036	.161	.812	.037	.289
		0.05	.804	.038	.195	.812	.039	.335
		0.1	.804	.039	.221	.812	.039	.354
	ML	0	.898	.016	.938	.899	.016	.939
		0.05	.898	.016	.941	.899	.016	.940
		0.1	.898	.017	.945	.899	.017	.939
non-tau-equivalent	Deletion	0	.670	.062	.679	.695	.059	.733
		0.05	.670	.065	.714	.694	.062	.764
		0.1	.669	.067	.746	.693	.063	.778
	ML	0	.772	.036	.935	.787	.034	.929
		0.05	.772	.037	.938	.787	.035	.934
		0.1	.771	.038	.938	.787	.036	.935

Note. Est: estimate; s.e.: Standard error; Coverage: coverage rate of 95% confidence intervals.

package includes an example data set with 100 cases and 10 variables. In this section, we illustrate the use of the R package and its online interface using this data set so that users can practice and replicate our analysis.

R Package coefficientalpha

To use the package within R, first install it using the command `install.pacakges('coefficientalpha')` and then load it using the command `library(coefficientalpha)`. The R package also includes tests of tau-equivalence[2] and homogeneity of items.[3] Both tests are based on the robust F statistic studied in Tong et al. (2014) and proposed by Yuan and Zhang (2012). The R input and output for the tests are given in Figure 2. Note that the numbers on the right of the figure are the line numbers to facilitate the explanation of the R code, not a part of the code itself. The code on Line 2 loads the example data into R for use. On Line 3, the function `tau.test` is used to carry out the tests. The output of the analysis is given from Line 6 to Line 12. First, the robust F statistic for the test of tau-equivalence

[2]The tau-equivalent test evaluates whether a one-factor model with equal factor loadings adequately fits the data.

[3]The test of homogeneity evaluates whether a one-factor model with freely estimated factor loadings adequately fits the data.

is 1.908 (Line 7) with a p-value 0.0114 (Line 8). Therefore, we have to reject the tau-equivalence assumption. However, the F statistic for the test of homogeneity is 1.401 with a p-value 0.1196, failing to reject the homogeneity assumption.

```
1  ### Input ###
2  data(example)
3  tau.test(example)
4
5  ### Output ###
6  Test of tau equivalent
7  The robust F statistic is  1.908
8  with a p-value  0.0114
9
10 Test of homogeneous items
11 The robust F statistic is  1.401
12 with a p-value  0.1196
```

Figure 2. Testing tau-equivalence and homogeneity of items

Because we rejected the tau-equivalence but failed to reject the homogeneity assumption, we proceed with estimating the robust omega in the following. The same procedure works for the estimation of robust alpha. The R code for the analysis as well as the output are given in Figure 3.

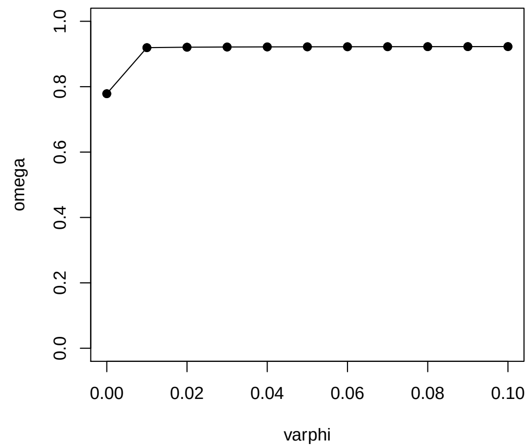
In Figure 3, Line 2 uses the function `omega` to initially obtain the robust estimate of coefficient omega corresponding to the downweighting rate `varphi=.1`. Note that if the downweighting rate is set at 0, the conventional non-robust omega will be calculated. The initial calculation is used for the diagnostic purpose and a relative large downweighting rate is usually specified. Line 3 uses the `plot` function to generate the diagnostic plot by specifying the plot type through `type='d'`. For the example data, the diagnostic plot is shown in Figure 4a, which plots the estimated omega corresponding to φ from 0 to 0.1 with an interval of 0.01. As φ increases, the estimate of omega first increases and then flattens out when $\varphi = 0.02$.[4] This indicates that 0.02 is a good choice for the downweighting rate. Therefore, for the rest of the analysis of the data, we let $\varphi = 0.02$. Line 4 calculates the robust omega by setting $\varphi = 0.02$. The standard error is requested by setting `se=TRUE`. The output on Lines 11-12 shows that the estimated robust omega is 0.921 with a robust standard error 0.020. Note that when $\varphi = 0$, the non-robust omega is 0.779. To get the 95% confidence interval for omega, the `summary` function on Line 5 is used with `prob=.95`. The output on Line 19 gives the confidence interval [0.882, 960].

[4]One can also use $\varphi = 0.01$ here. We use $\varphi = 0.02$ because the weight plot shows an additional possible outlying observation (Case 94).

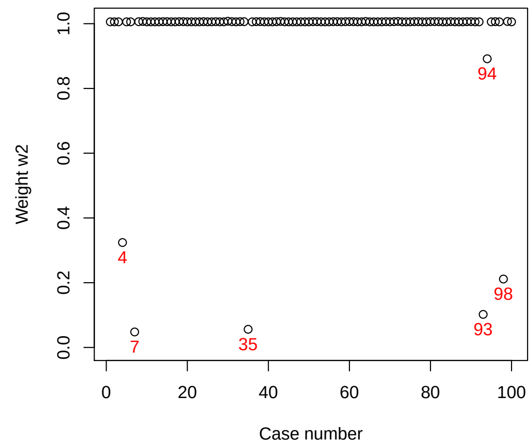
```
1 ### Input ###
2 omega.est<-omega(example, varphi=.1)
3 plot(omega.est, type='d')
4 omega.res<-omega(example, varphi=.02, se=TRUE)
5 summary(omega.res, prob=.95)
6 plot(omega.res, type='w', profile=6)
7 plot(omega.res, type='p', profile=6)
8
9 ### Output ###
10 > omega.res<-omega(example, varphi=.02, se=TRUE)
11 The omega is 0.9207303 with the standard error 0.01994788.
12 About 6% of cases were downweighted.
13 > summary(omega.res, prob=.95)
14
15 The estimated omega is
16   omega                      0.921
17   se                         0.020
18   p-value (omega>0)          0.000
19   Confidence interval        0.882       0.960
```

Figure 3. R code and output for robust omega estimation

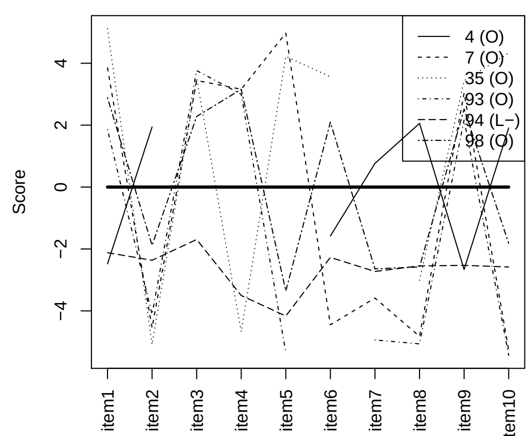
Lines 6 and 7 generate the weight plot (Figure 4b) and profile plot (Figure 4c), respectively. The option, `profile=6` is used to ask the software to label 6 cases because by default the software only identifies the 5 most influential cases. According to the weight plot in Figure 4b, 6 cases with ID 4, 7, 35, 93, 94, and 98 are downweighted. Except for Case 94, whose weight is about 0.9, each of the other 5 cases has a weight smaller than 0.4, indicating that these 5 cases would have the greatest influence in estimating coefficients omega when using the conventional non-robust method. In Figure 4c of the profile plot, each variable is centered at the robust estimate of the corresponding mean and plotted against the order of the variable. The profile plot suggests that, for Case 94, it has smaller values than average on all items. Therefore, Case 94 can be viewed as a leverage observation. For Cases 4, 7, 35, 93, and 98, they show the similar pattern where some items have values much larger than average while other items much smaller than average. Therefore, those cases can be viewed as outliers. Note the `plot` function clearly identifies outlier and leverage observation in the plot by using different legends.



(a) Diagnostic plot



(b) Weight plot



(c) Profile plot

Figure 4. The diagnostic, weight, and profile plots for estimating robust omega. Note O in the legend indicates an outlier. L+ indicates a leverage observation above average and L- indicates a leverage observation below average.

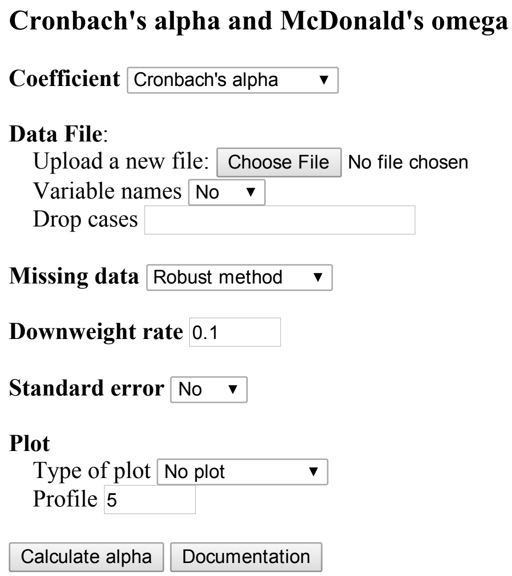


Figure 5. The online interface of the software coefficientalpha

Use the Online Software

For researchers who may not be familiar with R, a self-explanatory online interface is also developed to estimate robust alpha and omega. In particular, the use of the online interface does not require previous knowledge of R. To use the online interface, open the web page `http://psychstat.org/alpha` in a web browser. Figure 5 is a display of the online software interface. To use it, one first uploads a data file in free format (a text file with observations separated by space) with missing data denoted by "NA". Each column of the data file represents an item. Variable names can be specified in the first line of the data file. If a user prefers to remove certain cases instead of downweighting them, the case numbers can be provided. A user can choose to deal with missing data using our robust algorithm or just removing missing data through listwise deletion. The default value for the tuning parameter φ is 0.1, which can be modified. If setting $\varphi = 0$, then coefficient alpha will be estimated using the normal-based-ML (NML). A user can choose whether or not to estimate standard error for alpha. It is recommended but can be slow for large data sets. Finally, a user can choose to generate the three plots discussed in the paper. For the weight and profile plots, a user can specify how many cases to be labeled and highlighted and the default is 5.

Discussion

In using a test or scale, its reliability is always a primary concern. Cronbach's alpha is a widely used measure of reliability in the literature. When items are non-tau-equivalent, coefficient omega has been proposed and recommended. Methods for estimating the coefficient alpha and omega are commonly based on the sample covariance matrices or the NML estimates of Σ and, therefore, are not robust to outlying observations. In this study, we proposed a robust procedure to estimate alpha and omega as well as their confidence intervals. An R package coefficientalpha together with an online interface is also developed to carry out the proposed procedure.

With outlying observations, alpha and omega can be overestimated or underestimated. Our purpose is not to obtain a larger estimate but one that is closer to the population value without being overwhelmingly affected by a few influential observations. Because alpha and omega commonly estimated using NML are unduly affected by outlying observations, they tend to perform poorly when validating scales across groups (e.g, gender, race, culture). In contrast, robust estimates of alpha or omega will still perform well even if small percentages of the participants in different groups endorse the items differently. Even if the majority of the participants in different groups endorse the scale differently, the robust alpha and omega can still tell the difference, since they are decided by the majority of the observations as reflected by Equation 3, where only a small percentage of cases lying far from the center are downweighted.

In practice, the mechanisms that lead to outlying observations can be more complex than discussed in the current study. For example, outlying observations can result from different understanding of a scale. It is well known that cultural differences exist between Americans and Chinese in the circumstances evoking pride, shame, and guilt (e.g., Stipek, 1998). Chinese may view expressing pride in public as an improper or unacceptable behavior, contrary to most Americans. Consequently, the recorded data for a Chinese participant in a study with a majority of American participants may be identified as outlying. Although such data might not be viewed as admissible anymore, our robust method can still be applied to identify the participant as outlying and further action, e.g., removing such observations, can be taken by a researcher with substantial knowledge of the study. For this purpose, our R package facilitates the identification of peculiar observations and allows a user to drop potentially erroneous ones. On the other hand, if a large number of participants show such a cultural difference, the population cannot be viewed as homogeneous any more. Then, multiple group models or mixture models might be needed for better analysis. Our robust method can be applied as a diagnostic tool to explore such heterogeneous samples.

There are also situations where non-normality is expected and one would not want to downweight the outlying observations. For example, data on abnormal behaviors in clinical research tend to be skewed and extreme scores in such data are often what a researcher is interested in. In this situation, one can opt not to downweight the extreme scores by setting $\varphi = 0$ in our R package. However, even for skewed data with heavy tails, robust estimate of alpha and omega may still be more accurate than the traditional ones based on the sample covariance matrix. In particular, the graphs of the R package allow users to identify the extreme observations and to examine their profiles as well as to explore how their inclusion or removal affects the evaluation of reliability.

In summary, practical data are often nonnormally distributed. When the mechanism of nonnormality is not clear, it is always a safer bet to utilize the robust reliability measures than their non-robust counterparts. At the same time, once outlying observations are identified, special attention might be paid.

In addition to coefficients alpha and omega, other reliability measures have also been proposed in the literature. They include, among others, coefficient β (Revelle & Zinbarg, 2009; Zinbarg et al., 2005), dimension-free and model-based internal consistency reliability (Bentler, 2009, 2010), and model-based reliability that allows nonlinear relationship (Green & Yang, 2009b; Yang & Green, 2010). In assessing inter-rater agreement, reliability coefficients can also be formulated as a form of intraclass correlation coefficients (Shrout & Fleiss, 1979). Under certain conditions, these alternative measures can be better estimates of reliability than alpha and omega. However, these reliability measures, typically calculated as functions of the sample covariance matrix, are also influenced by non-normal and missing data. Future study can investigate how to extend our robust procedure to these reliability measures. Furthermore, we will compare our robust method with the existing methods such as the bootstrap method and the L-moment method in the future.

Appendix A. Standard Error of $\hat{\alpha}$

Notice that $\hat{\boldsymbol{\Sigma}}$ is a symmetric matrix. Let $\hat{\boldsymbol{\sigma}}$ be the vector of nonduplicated elements of $\hat{\boldsymbol{\Sigma}}$. Then, under a set of regularity conditions, $\hat{\boldsymbol{\sigma}}$ is asymptotically normally distributed, which can be written as

$$\sqrt{n}(\hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}) \xrightarrow{\mathcal{L}} N(0, \boldsymbol{\Gamma}), \tag{6}$$

where $\boldsymbol{\sigma}$ is the population counterpart of $\hat{\boldsymbol{\sigma}}$, $\boldsymbol{\Gamma}$ is the asymptotic covariance matrix of $\sqrt{n}\hat{\boldsymbol{\sigma}}$, and can be consistently estimated by a sandwich-type covariance matrix $\hat{\boldsymbol{\Gamma}}$ (see Yuan & Zhang, 2012a). Since α is a function of $\boldsymbol{\sigma}$, it follows from the delta method that

$$\sqrt{n}(\hat{\alpha} - \alpha) \xrightarrow{\mathcal{L}} N(0, \omega^2), \tag{7}$$

where ω^2 can be consistently estimated by

$$\hat{\omega}^2 = \frac{\partial \hat{\alpha}}{\partial \hat{\boldsymbol{\sigma}}'} \hat{\boldsymbol{\Gamma}} \frac{\partial \hat{\alpha}}{\partial \hat{\boldsymbol{\sigma}}}. \tag{8}$$

It follows from Equation 1 that the elements of the vector $\partial\hat{\alpha}/\partial\hat{\boldsymbol{\sigma}}$ of partial derivatives can be evaluated according to

$$\frac{\partial \alpha}{\partial \sigma_{ij}} = \begin{cases} -\dfrac{p}{p-1}\left[\dfrac{1}{\sum_{i=1}^{p}\sum_{j=1}^{p}\sigma_{ij}} - \dfrac{\sum_{i=1}^{p}\sigma_{ii}}{(\sum_{i=1}^{p}\sum_{j=1}^{p}\sigma_{ij})^2}\right], & i = j \\ \dfrac{2p}{p-1}\left[\dfrac{\sum_{i=1}^{p}\sigma_{ii}}{(\sum_{i=1}^{p}\sum_{j=1}^{p}\sigma_{ij})^2}\right], & i \neq j \end{cases}. \tag{9}$$

SE for $\hat{\alpha}$ and the corresponding CI as implemented in the package coefficientalpha are calculated according to the asymptotic distribution in Equation 7 and the variance estimate in Equation 8.

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