

# Package ‘ctmcd’

July 22, 2025

**Type** Package

**Title** Estimating the Parameters of a Continuous-Time Markov Chain from Discrete-Time Data

**Version** 1.4.4

**Date** 2024-02-07

## Description

Estimation of Markov generator matrices from discrete-time observations. The implemented approaches comprise diagonal and weighted adjustment of matrix logarithm based candidate solutions as in Israel (2001) <doi:10.1111/1467-9965.00114> as well as a quasi-optimization approach. Moreover, the expectation-maximization algorithm and the Gibbs sampling approach of Bladt and Sorensen (2005) <doi:10.1111/j.1467-9868.2005.00508.x> are included.

**License** GPL-3

**Imports** Rcpp (>= 1.0.12), coda, expm, numDeriv

**Suggests** knitr, rmarkdown, R.rsp

**LinkingTo** Rcpp, RcppArmadillo

**VignetteBuilder** knitr, R.rsp

**NeedsCompilation** yes

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**Repository** CRAN

**Date/Publication** 2024-02-09 09:40:02 UTC

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ctmcd-package	<i>Estimating the Parameters of a Continuous-Time Markov Chain from Discrete-Time Data</i>
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## Description

Functions for estimating Markov generator matrices from discrete-time observations.

## Author(s)

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## References

- M. Pfeuffer: ctmcd: An R Package for Estimating the Parameters of a Continuous-Time Markov Chain from Discrete-Time Data. *The R Journal* 9(2):127-141, 2017
- M. Pfeuffer. Generator Matrix Approximation Based on Discrete-Time Rating Migration Data. Master Thesis, Ludwig Maximilian University of Munich, 2016
- R. B. Israel et al.: Finding Generators for Markov Chains via Empirical Transition Matrices, with Applications to Credit Ratings. *Mathematical Finance* 11(2):245-265, 2001
- E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. *Algo Research Quarterly* 4(1):23-40, 2001
- M. Bladt and M. Soerensen: Statistical Inference for Discretely Observed Markov Jump Processes. *Journal of the Royal Statistical Society B* 67(3):395-410, 2005

**Examples**

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)
plot(gmem)

## Confidence Interval
ciem=ginci(gmem,alpha=0.05)
plot(ciem)
```

---

ctmcdlogLik

*Discrete-Time Data Log-Likelihood Function*

---

**Description**

Function for evaluating the likelihood function of a continuous-time Markov chain given discrete-time data.

**Usage**

```
ctmcdlogLik(gm, tmabs, te)
```

**Arguments**

gm	generator matrix of continuous-time Markov chain
tmabs	matrix of absolute transition frequencies
te	time elapsed in transition process

**Value**

log-likelihood value

**Author(s)**

Marius Pfeuffer

**Examples**

```

data(tm_abs)

## Initial guess for generator matrix (absorbing default state)
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

## Log-likelihood of initial guess
ctmcdlogLik(gm0,tm_abs,1)

```

---

<code>gm</code>	<i>Generator Matrix Estimation</i>
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---

**Description**

Generic function to estimate the parameters of a continuous Markov chain

**Usage**

```
gm(tm, te, method, ...)
```

**Arguments**

<code>tm</code>	matrix of either absolute transition frequencies (if method is "EM" or "GS") or relative transition frequencies (if method is "DA", "WA" or "QO")
<code>te</code>	time elapsed in transition process
<code>method</code>	method to derive generator matrix: "DA" - Diagonal Adjustment, "WA" - Weighted Adjustment, "QO" - Quasi-Optimization, "EM" - Expectation-Maximization Algorithm, "GS" - Gibbs Sampler
<code>...</code>	Additional Arguments: <ul style="list-style-type: none"> <li>• <code>gmguess</code>: initial guess for generator matrix estimation procedure (if method is "EM")</li> <li>• <code>prior</code>: prior parametrization (if method is "GS")</li> <li>• <code>burnin</code>: burn-in period (if method is "GS")</li> <li>• <code>eps</code>: convergence criterion (if method is "EM")</li> <li>• <code>conv_pvalue,conv_freq</code>: convergence criterion (if method is "GS")</li> <li>• <code>niter</code>: maximum number of iterations (if method is "EM" or "GS")</li> <li>• <code>sampl_func</code>: optional self-written path sampling function for endpoint-conditioned Markov processes (if method is "GS")</li> <li>• <code>compmat</code>: matrix stating combined use of modified rejection sampling / uniformization sampling algorithms (if method is "GS")</li> <li>• <code>sampl_method</code>: sampling method for deriving endpoint-conditioned Markov process path: "Unif" - Uniformization Sampling, "ModRej" - Modified Rejection Sampling (if method is "GS")</li> </ul>

- logmethod: method to compute matrix logarithm (if method is "DA", "WA" or "QO", see ?logm from expm package for more information)
- expmethod: method to compute matrix exponential (if method is "EM" or "GS", see ?expm from expm package for more information)
- verbose: verbose mode (if method is "EM" or "GS")

### Details

The methods "DA", "WA" and "QO" provide adjustments of a matrix logarithm based candidate solution, "EM" gives the maximum likelihood estimate and "GS" a posterior mean estimate in a Bayesian setting with conjugate Gamma priors.

### Value

generator matrix estimate

### Author(s)

Marius Pfeuffer

### References

G. dos Reis, M. Pfeuffer, G. Smith: Capturing Rating Momentum in the Estimation of Probabilities of Default, With Application to Credit Rating Migrations (In Preparation), 2018

M. Pfeuffer: Generator Matrix Approximation Based on Discrete Time Rating Migration Data. Master Thesis, University of Munich, 2016

Y. Inamura: Estimating Continuous Time Transition Matrices from Discretely Observed Data. Bank of Japan Working Paper Series, 2006

R. B. Israel et al.: Finding Generators for Markov Chains via Empirical Transition Matrices, with Applications to Credit Ratings. Mathematical Finance 11(2):245-265, 2001

E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. Algo Research Quarterly 4(1):23-40, 2001

M. Bladt and M. Soerensen: Statistical Inference for Discretely Observed Markov Jump Processes. Journal of the Royal Statistical Society B 67(3):395-410, 2005

### See Also

[gmDA](#), [gmWA](#), [gmQO](#), [gmEM](#), [gmGS](#)

### Examples

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0
```

```

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)
gmem

## Quasi Optimization Estimate
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))

gmqo=gm(tm_rel,te=1,method="QO")
gmqo

```

---

gm.default

*Generator Matrix Estimation*


---

## Description

Default function to estimate the parameters of a continuous Markov chain

## Usage

```

## Default S3 method:
gm(tm, te, method, gmguess = NULL, prior = NULL, burnin = NULL,
eps = 1e-06, conv_pvalue = 0.05, conv_freq = 10, niter = 10000, sampl_func = NULL,
combat = NULL, sampl_method = "Unif", logmethod = "Eigen", expmethod = "PadeRBS",
verbose = FALSE, ...)

```

## Arguments

tm	matrix of either absolute transition frequencies (if method is "EM" or "GS") or relative transition frequencies (if method is "DA", "WA" or "QO")
te	time elapsed in transition process
method	method to derive generator matrix: "DA" - Diagonal Adjustment, "WA" - Weighted Adjustment, "QO" - Quasi-Optimization, "EM" - Expectation-Maximization Algorithm, "GS" - Gibbs Sampler
gmguess	initial guess for generator matrix estimation procedure (if method is "EM")
prior	prior parametrization (if method is "GS")
burnin	burn-in period (if method is "GS")
eps	convergence criterion (if method is "EM" or "GS")
conv_pvalue	convergence criterion: stop, if Heidelberger and Welch's diagnostic assumes convergence (see coda package)
conv_freq	convergence criterion: absolute frequency of convergence evaluations
niter	maximum number of iterations (if method is "EM" or "GS")
sampl_func	optional self-written path sampling function for endpoint-conditioned Markov processes (if method is "GS")
combat	matrix stating combined use of modified rejection sampling / uniformization sampling algorithms (if method is "GS")

sampl_method	sampling method for deriving endpoint-conditioned Markov process path: "Unif" - Uniformization Sampling, "ModRej" - Modified Rejection Sampling (if method is "GS")
logmethod	method to compute matrix logarithm (if method is "DA", "WA" or "QO", see ?logm from expm package for more information)
expmethod	method to compute matrix exponential (if method is "EM" or "GS", see ?expm from expm package for more information)
verbose	verbose mode (if method is "EM" or "GS")
...	additional arguments

### Details

The methods "DA", "WA" and "QO" provide adjustments of a matrix logarithm based candidate solution, "EM" gives the maximum likelihood estimate and "GS" a posterior mean estimate in a Bayesian setting with conjugate Gamma priors.

### Value

generator matrix estimate

### Author(s)

Marius Pfeuffer

### References

- M. Pfeuffer: Generator Matrix Approximation Based on Discrete Time Rating Migration Data. Master Thesis, University of Munich, 2016
- Y. Inamura: Estimating Continuous Time Transition Matrices from Discretely Observed Data. Bank of Japan Working Paper Series, 2006
- R. B. Israel et al.: Finding Generators for Markov Chains via Empirical Transition Matrices, with Applications to Credit Ratings. *Mathematical Finance* 11(2):245-265, 2001
- E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. *Algo Research Quarterly* 4(1):23-40, 2001
- M. Bladt and M. Soerensen: Statistical Inference for Discretely Observed Markov Jump Processes. *Journal of the Royal Statistical Society B* 67(3):395-410, 2005

### See Also

[gmDA](#), [gmWA](#), [gmQO](#), [gmEM](#), [gmGS](#)

### Examples

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
```

```
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)
gmem

## Quasi Optimization Estimate
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))

gmqo=gm(tm_rel,te=1,method="QO")
gmqo
```

---

gmci

*Confidence / Credibility Intervals for Generator Matrix Objects*


---

### Description

Generic function to derive confidence / credibility intervals for "EM" or "GS" based generator matrix objects

### Usage

```
gmci(gm, alpha, ...)
```

### Arguments

gm	a "EM" or "GS" generator matrix object
alpha	significance level
...	additional arguments: <ul style="list-style-type: none"> <li>• eps: threshold for which generator matrix parameters are assumed to be fixed at zero (if "EM" object)</li> <li>• cimethod: "Direct" and "SdR" use analytical expressions of the Fisher information matrix, "BS" employs the numerical approach of Bladt and Sorensen, 2009 (if "EM" object)</li> <li>• expmethod: method to compute matrix exponentials (see ?expm from expm package for more information)</li> </ul>

### Details

If gm is based on the "EM" method (expectation-maximization algorithm), the function computes a Wald confidence interval based on the method of Oakes, 1999. IF gm is based on the "GS" method (Gibbs sampler), the function computes an equal-tailed credibility interval.

### Value

generator matrix confidence bounds



**Author(s)**

Marius Pfeuffer

**References**

M. Bladt and M. Soerensen. Efficient Estimation of Transition Rates Between Credit Ratings from Observations at Discrete Time Points. *Quantitative Finance*, 9(2):147-160, 2009

D. Oakes. Direct calculation of the information matrix via the EM algorithm. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 61(2):479-482, 1999

G. Smith and G. dos Reis. Robust and Consistent Estimation of Generators in Credit Risk. *Quantitative Finance* 18(6):983-1001, 2018

G. dos Reis, M. Pfeuffer, G. Smith: Capturing Rating Momentum in the Estimation of Probabilities of Default, With Application to Credit Rating Migrations (In Preparation), 2018

**Examples**

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)

## Oakes Confidence Interval
ciem=gmci(gmem,alpha=0.05)
ciem
```

---

gmci.default

*Confidence / Credibility Intervals for Generator Matrix Objects*


---

**Description**

Default function to derive confidence / credibility intervals for "EM" or "GS" based generator matrix objects

**Usage**

```
## Default S3 method:
gmci(gm, alpha, eps = 1e-04, cimethod="Direct", expmethod = "PadeRBS", ...)
```

**Arguments**

gm	a "EM" or "GS" generator matrix object
alpha	significance level
eps	threshold for which generator matrix parameters are assumed to be fixed at zero (if "EM" object)
cimethod	"Direct" or "SdR" use analytical expressions of the Fisher information matrix, "BS" employ the numerical expressions of Bladt and Soerensen, 2009 (if "EM" object)
expmethod	method to compute matrix exponentials (see ?expm from expm package for more information)
...	additional arguments

**Details**

If gm is based on the "EM" method (expectation-maximization algorithm), the function computes a Wald confidence interval based on the method of Oakes, 1999. If gm is based on the "GS" method (Gibbs sampler), the function computes an equal-tailed credibility interval.

**Value**

generator matrix confidence bounds

**Author(s)**

Marius Pfeuffer

**References**

G. dos Reis, M. Pfeuffer, G. Smith: Capturing Rating Momentum in the Estimation of Probabilities of Default, With Application to Credit Rating Migrations (In Preparation), 2018

**Examples**

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)

## Oakes Confidence Interval
ciem=gmci(gmem,alpha=0.05)
ciem
```

---

gmDA

*Diagonal Adjustment*

---

### Description

Function for deriving a Markov generator matrix estimate based on the diagonal adjustment method of Israel et al., 2001

### Usage

```
gmDA(tmrel, te, logmethod = "Eigen")
```

### Arguments

tmrel	matrix of relative transition frequencies
te	time elapsed in transition process
logmethod	method for computation of matrix logarithm, by default eigendecomposition is chosen (see ?logm from expm package for more information)

### Details

A candidate solution is derived by the matrix logarithm and then adjusted in order to fulfil the properties of a Markov generator matrix.

### Value

generator matrix estimate

### Author(s)

Marius Pfeuffer

### References

R. B. Israel et al.: Finding Generators for Markov Chains via Empirical Transition Matrices, with Applications to Credit Ratings. *Mathematical Finance* 11(2):245-265, 2001

### Examples

```
## Derive matrix of relative transition frequencies
data(tm_abs)
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))

## Derive diagonal adjustment generator matrix estimate
gmda=gmDA(tm_rel,1)
gmda
```

---

gmEM

*Expectation-Maximization Algorithm*

---

### Description

Function for deriving a Markov generator matrix estimate by an instance of the expectation-maximization algorithm (described by Bladt and Soerensen, 2005)

### Usage

```
gmEM(tmabs, te, gmguess, eps = 1e-06, niter = 10000, expmethod = "PadeRBS",  
      verbose = FALSE)
```

### Arguments

tmabs	matrix of absolute transition frequencies
te	time elapsed in transition process
gmguess	initial guess (for generator matrix)
eps	stop criterion: stop, if relative change in log-likelihood is smaller than eps
niter	stop criterion: maximum number of iterations
expmethod	method for computation of matrix exponential, by default "PadeRBS" is chosen (see ?expm from expm package for more information)
verbose	verbose mode

### Details

A maximum likelihood generator matrix estimate is derived by an instance of the expectation-maximization algorithm.

### Value

generator matrix estimate

### Author(s)

Marius Pfeuffer

### References

M. Bladt and M. Soerensen: Statistical Inference for Discretely Observed Markov Jump Processes. Journal of the Royal Statistical Society B 67(3):395-410, 2005

**Examples**

```

data(tm_abs)

## Initial guess for generator matrix (absorbing default state)
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

## Derive expectation-maximization algorithm generator matrix estimate
gmem=gmEM(tmabs=tm_abs,1,gmguess=gm0,verbose=TRUE)
gmem

```

gmGS

*Gibbs Sampler***Description**

Function for deriving a Markov generator matrix estimate by Gibbs sampling (described by Bladt and Soerensen, 2005)

**Usage**

```

gmGS(tmabs, te, prior, burnin, conv_pvalue = 0, conv_freq = 10,
niter = 10000, sampl_method = "Unif", expmethod = "PadeRBS", verbose = FALSE,
combat=NULL, sampl_func = NULL)

```

**Arguments**

tmabs	matrix of absolute transition frequencies
te	time elapsed in transition process
prior	list of prior parameters (Gamma prior)
burnin	number of burn-in iterations
conv_pvalue	convergence criterion: stop, if Heidelberger and Welch's diagnostic assumes convergence (see coda package), convergence check is only employed if conv_pvalue>0
conv_freq	convergence criterion: absolute frequency of convergence evaluations
niter	stop criterion: stop, if maximum number of iterations is exceeded
sampl_method	method for sampling paths from endpoint-conditioned Markov processes. options: "Unif" - Uniformization sampling, "ModRej" - Modified Rejection Sampling
expmethod	method for computation of matrix exponential, by default "PadeRBS" is chosen (see ?expm from expm package for more information)
verbose	verbose mode
combat	matrix specifying the combined use of sampling methods: "U" - uniformization sampling, "M" - modified rejection sampling
sampl_func	interface for own endpoint-conditioned Markov process sampling function

**Details**

A posterior mean generator matrix estimate is derived by Gibbs Sampling. The gamma distribution is used as prior.

**Value**

generator matrix estimate

**Author(s)**

Marius Pfeuffer

**References**

M. Bladt and M. Soerensen: Statistical Inference for Discretely Observed Markov Jump Processes. Journal of the Royal Statistical Society B 67(3):395-410, 2005

**See Also**

[rNijTRiT\\_ModRej](#), [rNijTRiT\\_Unif](#)

**Examples**

```
data(tm_abs)

## Example prior parametrization (absorbing default state)
pr=list()
pr[[1]]=matrix(1,8,8)
pr[[1]][8,]=0

pr[[2]]=c(rep(5,7),Inf)

## Derive Gibbs sampling generator matrix estimate

gmgs=gmGS(tmabs=tm_abs,te=1,sampl_method="Unif",prior=pr,burnin=10,niter=100,verbose=TRUE)
gmgs
```

**Description**

Function for deriving a Markov generator matrix estimate based on the quasi-optimization procedure of Kreinin and Sidelnikova, 2001

**Usage**

```
gmQO(tmrel, te, logmethod = "Eigen")
```

**Arguments**

tmrel	matrix of relative transition frequencies
te	time elapsed in transition process
logmethod	method for computation of matrix logarithm, by default eigendecomposition is chosen (see ?logm from expm package for more information)

**Details**

From the set of possible Markov generator matrices, the one is chosen which is closest to a matrix logarithm based candidate solution in terms of sum of squared deviations.

**Value**

generator matrix estimate

**Author(s)**

Marius Pfeuffer

**References**

E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. *Algo Research Quarterly* 4(1):23-40, 2001

**Examples**

```
data(tm_abs)
## Derive matrix of relative transition frequencies
data(tm_abs)
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))

## Derive quasi optimization generator matrix estimate
gmqo=gmQO(tm_rel,1)
gmqo
```

---

gmWA

*Weighted Adjustment*


---

**Description**

Function for deriving a Markov generator matrix estimate based on the weighted adjustment method of Israel et al., 2001

**Usage**

```
gmWA(tmrel, te, logmethod = "Eigen")
```

**Arguments**

tmrel	matrix of relative transition frequencies
te	time elapsed in transition process
logmethod	method for computation of matrix logarithm, by default eigendecomposition is chosen (see ?logm from expm package for more information)

**Details**

A candidate solution is derived by the matrix logarithm and then adjusted in order to fulfil the properties of a Markov generator matrix.

**Value**

generator matrix estimate

**Author(s)**

Marius Pfeuffer

**References**

R. B. Israel et al.: Finding Generators for Markov Chains via Empirical Transition Matrices, with Applications to Credit Ratings. *Mathematical Finance* 11(2):245-265, 2001

**Examples**

```
## Derive matrix of relative transition frequencies
data(tm_abs)
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))

## Derive weighted adjustment generator matrix estimate
gmwa=gmWA(tm_rel,1)
gmwa
```

---

plot.gm

*Plot Function for Generator Matrix Estimation Objects*

---

**Description**

Function for visualizing the output of a generator matrix estimation procedure.



**Usage**

```
## S3 method for class 'gm'
plot(x, mattext, col = c("grey", "red"), main = x$method, las = 1,
     xlab = "To", ylab = "From", xnames, ynames, cex = 1, fig = 3, opacity_factor, ...)
```

**Arguments**

x	a generator matrix estimation object
mattext	optional: matrix of strings replacing the parameter estimates
col	two element vector of basis colors for positive and negative parameter estimate entries
main	optional: plot title
las	orientation of x and y axis elements
xlab	x axis name
ylab	y axis name
xnames	description of x axis elements
ynames	description of y axis elements
cex	font size
fig	number of significant figure to be plotted
opacity_factor	two element vector for specification of opacity for positive and negative parameter entry highlighting (must be greater than zero)
...	additional arguments

**Value**

no value, plot function

**Author(s)**

Marius Pfeuffer

**See Also**

[print.gm](#), [summary.gm](#), [plotM](#)

**Examples**

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs, te=1, method="EM", gmguess=gm0)
plot(gmem)
```

---

plot.gmci	<i>Plot Function for Generator Matrix Confidence / Credibility Interval Objects</i>
-----------	---

---

**Description**

Function for visualizing the boundaries of generator matrix confidence / credibility intervals

**Usage**

```
## S3 method for class 'gmci'
plot(x, mattext, col = c("grey", "red"), main, las = 1, xlab = "To",
     ylab = "From", xnames, ynames, cex = 1, fig = 2, opacity_factor, ...)
```

**Arguments**

x	a generator matrix confidence / credibility interval object
mattext	optional: matrix of strings replacing the parameter estimates
col	two element vector of basis colors for positive and negative parameter estimate entries
main	optional: plot title
las	orientation of x and y axis elements
xlab	x axis name
ylab	y axis name
xnames	description of x axis elements
ynames	description of y axis elements
cex	font size
fig	number of significant figures to be plotted
opacity_factor	two element vector for specification of opacity for positive and negative parameter entry highlighting (must be greater than zero)
...	additional arguments

**Value**

no value, plot function

**Author(s)**

Marius Pfeuffer

**See Also**

[print.gmci](#), [plotM](#)

**Examples**

```

data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)
plot(gmem)

## Confidence Interval
ciem=gmci(gmem,alpha=0.05)
plot(ciem)

```

plotM

*Matrix Plot Function***Description**

Function to visualize matrices

**Usage**

```

plotM(mat, mattext, col = c("grey", "red"), main, las = 1, xlab = "To",
ylab = "From", xnames, ynames, cex = min(1, nrow(mat)/8), fig = 3, opacity_factor)

```

**Arguments**

mat	a matrix
mattext	optional: matrix of strings replacing the original matrix entries
col	two element vector of basis colors for positive and negative matrix entries
main	optional: plot title
las	orientation of x and y axis elements
xlab	x axis name
ylab	y axis name
xnames	description of x axis elements
ynames	description of y axis elements
cex	font size
fig	number of significant figures to be plotted
opacity_factor	two element vector for specification of opacity for positive and negative parameter entry highlighting (must be greater than zero)

**Value**

no value, plot function

**Author(s)**

Marius Pfeuffer

**See Also**

[plot.gm](#), [plot.gmci](#)

**Examples**

```
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

plotM(gm0)
```

---

print.gm

*Print Method for Generator Matrix Estimation Objects*

---

**Description**

Function for printing the results of a generator matrix estimation

**Usage**

```
## S3 method for class 'gm'
print(x, ...)
```

**Arguments**

x	a generator matrix estimation object
...	additional arguments

**Value**

generator matrix

**See Also**

[summary.gm](#), [plot.gm](#)

---

print.gmci	<i>Print Method for Generator Matrix Confidence / Credibility Interval Objects</i>
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---

**Description**

Function for printing the boundaries of a generator matrix confidence / credibility interval

**Usage**

```
## S3 method for class 'gmci'
print(x, ...)
```

**Arguments**

x	a generator matrix confidence / credibility interval
...	additional arguments

**Value**

generator matrix confidence bounds

**See Also**

[plot.gmci](#)

---

rNijTRiT_ModRej	<i>C++ Based Modified Rejection Sampling</i>
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---

**Description**

Function for generating initial and endpoint-conditioned Markov process sampling paths for a given discrete-time transition matrix

**Usage**

```
rNijTRiT_ModRej(tmabs, te, gm)
```

**Arguments**

tmabs	matrix of absolute transition frequencies
te	time elapsed in transition process
gm	generator matrix

**Details**

Function for the simulation of paths from an endpoint-conditioned Markov process. Returns number of transitions NijT and cumulative holding times RiT.

**Value**

endpoint-conditioned sampling path

**Author(s)**

Jon Fintzi, Marius Pfeuffer

**References**

J. Fintzi: R Package ECctmc, 2016.

A. Hobolth and E. A. Stone: Simulation from Endpoint-Conditioned, Continuous-Time Markov Chains on a Finite State Space, with Applications to Molecular Evolution. *Annals of Applied Statistics* 3(3):1204-1231, 2009

**Examples**

```
data(tm_abs)

## Initial guess for generator matrix (absorbing default state)

gm=matrix(1,8,8)
diag(gm)=0
diag(gm)=-rowSums(gm)
gm[8,]=0

rNijTRiT_ModRej(tm_abs,1,gm)
```

---

rNijTRiT\_Unif

*C++ Based Uniformization Sampling*

---

**Description**

Function for generating initial and endpoint-conditioned Markov process sampling paths for a given discrete-time transition matrix

**Usage**

```
rNijTRiT_Unif(tmabs, te, gm, tpm)
```

**Arguments**

<code>tmabs</code>	matrix of absolute transition frequencies
<code>te</code>	time elapsed in transition process
<code>gm</code>	generator matrix
<code>tpm</code>	discrete-time transition probability matrix, matrix exponential of <code>gm</code>

**Details**

Function for the simulation of paths from an endpoint-conditioned Markov process. Returns number of transitions `NijT` and cumulative holding times `RiT`.

**Value**

endpoint-conditioned sampling path

**Author(s)**

Jon Fintzi, Marius Pfeuffer

**References**

J. Fintzi: R Package `ECctmc`, 2016.

A. Hobolth and E. A. Stone: Simulation from Endpoint-Conditioned, Continuous-Time Markov Chains on a Finite State Space, with Applications to Molecular Evolution. *Annals of Applied Statistics* 3(3):1204-1231, 2009

**Examples**

```
data(tm_abs)

## Generator Matrix
gm=matrix(1,8,8)
diag(gm)=0
diag(gm)=-rowSums(gm)
gm[8,]=0

## Transition Probability Matrix
library(expm)
te=1
tpm=expm(gm*te)

rNijTRiT_Unif(tm_abs,te,gm,tpm)
```

---

summary.gm

*Extended Output for Generator Matrix Estimate Objects*


---

**Description**

Function for providing results and extended output of a generator matrix estimation procedure.

**Usage**

```
## S3 method for class 'gm'
summary(object, ...)
```

**Arguments**

object	a generator matrix estimation object
...	additional arguments

**Value**

estimation summary

**See Also**

[print.gm](#), [plot.gm](#)

---

tmci

*Delta Method Confidence Intervals for Matrix Exponential Transformations of Generator Matrix Objects*


---

**Description**

Generic function to derive delta method based confidence intervals for matrix exponential transformations of "EM" based generator matrix objects

**Usage**

```
tmci(gmem, alpha, te, eps = 1e-04, expmethod = "PadeRBS")
```

**Arguments**

gmem	an "EM" generator matrix object
alpha	significance level
te	discrete time horizon for which the interval is supposed to be computed
eps	threshold for which generator matrix parameters are assumed to be fixed at zero
expmethod	method to compute matrix exponentials (see ?expm from expm package for more information)



**Details**

Confidence intervals for discrete-time transition matrix predictions given generator matrix estimates are computed by using the delta method for matrix exponential transformations.

**Value**

transition matrix confidence bounds

**References**

G. dos Reis, M. Pfeuffer, G. Smith: Capturing Rating Momentum in the Estimation of Probabilities of Default, With Application to Credit Rating Migrations (In Preparation), 2018

**Examples**

```
data(tm_abs)

## Maximum Likelihood Generator Matrix Estimate
gm0=matrix(1,8,8)
diag(gm0)=0
diag(gm0)=-rowSums(gm0)
gm0[8,]=0

gmem=gm(tm_abs,te=1,method="EM",gmguess=gm0)

## 2.5 Year Transition Matrix Confidence Interval
citm=tmci(gmem,alpha=0.05,te=2.5)
citm
```

---

tm\_abs

*Single Year Corporate Credit Rating Transitions*


---

**Description**

Matrix of Standard and Poor's Global Corporate Rating Transition Frequencies 2000 (NR Removed)

**Usage**

```
data("tm_abs")
```

**Format**

The format is: num [1:8, 1:8] 17 2 0 0 0 0 0 1 455 ... - attr(\*, "dimnames")=List of 2 ..\$: chr [1:8] "AAA" "AA" "A" "BBB" ... ..\$: chr [1:8] "AAA" "AA" "A" "BBB" ...

**References**

European Securities and Markets Authority, 2016  
<https://cerep.esma.europa.eu/cerep-web/statistics/transitionMatrice.xhtml>

**Examples**

```
data(tm_abs)

## Matrix of relative transition frequencies
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))
tm_rel
```

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